

Hong Kong Mathematics Olympiad (1996 – 97)

Final Event 1 (Group)

香港數學競賽 (1996 – 97)

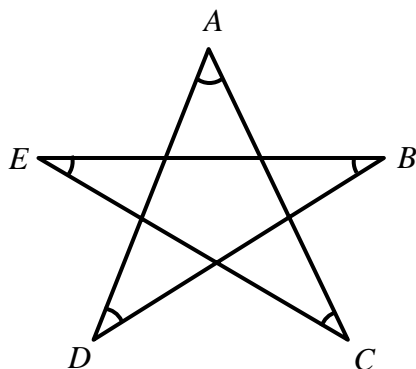
決賽項目 1 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) In the diagram $\angle A + \angle B + \angle C + \angle D + \angle E = a^\circ$. Find a .

$a =$

圖中， $\angle A + \angle B + \angle C + \angle D + \angle E = a^\circ$ 。求 a 。



- (ii) There are x terms in the algebraic expression $x^6 + x^6 + x^6 + \cdots + x^6$ and its sum is x^b . Find b .

$b =$

代數式 $x^6 + x^6 + x^6 + \cdots + x^6$ 有 x 項及其總和為 x^b 。求 b 。

- (iii) If $1 + 3 + 3^2 + 3^3 + \cdots + 3^8 = \frac{3^c - 1}{2}$, find c .

$c =$

若 $1 + 3 + 3^2 + 3^3 + \cdots + 3^8 = \frac{3^c - 1}{2}$ ，求 c 。

- (iv) 16 cards are marked from 1 to 16 and one is drawn at random. If the chance of it being a perfect square number is $\frac{1}{d}$, find d .

$d =$

從 16 張寫上 1 至 16 的咭紙中隨意抽出一張，若果抽出的號碼是一個完全平方數的概率為 $\frac{1}{d}$ ，求 d 之值。

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Final Event 2 (Group)

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決賽項目 2 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) If the sequence $1, 6 + 2a, 10 + 5a, \dots$ form an A.P., find a .

$a =$

若數列 $1, 6 + 2a, 10 + 5a, \dots$ 是一算術級數，求 a 。

- (ii) If $(0.0025 \times 40)^b = \frac{1}{100}$, find b .

$b =$

若 $(0.0025 \times 40)^b = \frac{1}{100}$ ，求 b 。

- (iii) If c is an integer and $c^3 + 3c + \frac{3}{c} + \frac{1}{c^3} = 8$, find c .

$c =$

若 c 為正整數及 $c^3 + 3c + \frac{3}{c} + \frac{1}{c^3} = 8$ ，求 c 。

- (iv) There are d different ways for arranging 5 girls in a row. Find d .

$d =$

若將 5 個女孩排成一行，共有 d 個不同方法。求 d 。

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Final Event 3 (Group)

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決賽項目 3 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) Let m be an integer satisfying the inequality

$$14x - 7(3x - 8) < 4(25 + x)$$

Find the least value of m .

設 m 為滿足不等式 $14x - 7(3x - 8) < 4(25 + x)$ 的整數，
求 m 的最小值。

$m =$

- (ii) It is given that $f(x) = \frac{1}{3}x^3 - 2x^2 + \frac{2}{3}x^3 + 3x^2 + 5x + 7 - 4x$. If $f(-2) = b$, find b .

已知 $f(x) = \frac{1}{3}x^3 - 2x^2 + \frac{2}{3}x^3 + 3x^2 + 5x + 7 - 4x$ 。若 $f(-2) = b$ ，求 b 。

$b =$

- (iii) It is given that $\log \frac{x}{2} = 0.5$ and $\log \frac{y}{5} = 0.1$. If $\log xy = c$, find c .

已知 $\log \frac{x}{2} = 0.5$ 及 $\log \frac{y}{5} = 0.1$ 。若 $\log xy = c$ ，求 c 。

$c =$

- (iv) Three prime numbers d , e and f which are all less than 10, satisfy the two conditions $d + e = f$ and $d < e$. Find d .

d 、 e 及 f 為三個小於 10 之質數且滿足兩個條件 $d + e = f$ 及 $d < e$ 。
求 d 。

$d =$

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Final Event 4 (Group)

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決賽項目 4 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) It is given that $a = 103 \times 97 \times 10009$, find a .

已知 $a = 103 \times 97 \times 10009$ ，求 a 。

$a =$

- (ii) It is given that $1 + x + x^2 + x^3 + x^4 = 0$.

If $b = 2 + x + x^2 + x^3 + x^4 + \cdots + x^{1989}$, find b .

已知 $1 + x + x^2 + x^3 + x^4 = 0$ 。若 $b = 2 + x + x^2 + x^3 + x^4 + \cdots + x^{1989}$ ，求 b 。

$b =$

- (iii) It is given that m and n are two natural numbers and both are not greater than 10. If c is the number of pairs of m and n satisfying the equation $mx = n$, where $\frac{1}{4} < x < \frac{1}{3}$, find c .

已知 m 及 n 為二個不大於 10 的自然數。若 c 為 m 及 n 滿足方程 $mx = n$ 之組數，其中 $\frac{1}{4} < x < \frac{1}{3}$ 。求 c 。

$c =$

- (iv) Let x and y be real numbers and define the operation $*$ as $x * y = px^y + q + 1$.

It is given that $1 * 2 = 869$ and $2 * 3 = 883$. If $2 * 9 = d$, find d .

設 x 及 y 為實數且定義運算 $*$ 為 $x * y = px^y + q + 1$ 。已知 $1 * 2 = 869$ 及 $2 * 3 = 883$ 。若 $2 * 9 = d$ ，求 d 。

$d =$

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Final Event 5 (Group)

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決賽項目 5 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) If a is a positive multiple of 5, which gives remainder 1 when divided by 3, find the smallest possible value of a .

若 a 是 5 的正整數倍，且被 3 除時餘 1，求 a 之最小可能數值。

$a =$

- (ii) If $x^3 + 6x^2 + 12x + 17 = (x + 2)^3 + b$, find b .

$b =$

若 $x^3 + 6x^2 + 12x + 17 = (x + 2)^3 + b$ ，求 b 。

- (iii) If c is a 2-digit positive integer such that the sum of its digits is 10 and the product of its digit is 25, find c .

$c =$

若 c 是一兩位正整數，且其兩位之和是 10，而其兩位之積是 25，求 c 。

- (iv) Let S_1, S_2, \dots, S_{10} be the first ten terms of an A.P., which consists of positive integers.

If $S_1 + S_2 + \dots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$, find d .

$d =$

設 S_1, S_2, \dots, S_{10} 是一個由正整數組成的 A.P. 之首 10 項。

若 $S_1 + S_2 + \dots + S_{10} = 55$ 及 $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$ ，求 d 。